Date of Examination: 20-Feb-2023   **Session (FN/AN):** AN
Duration: 2 hrs   **Full marks:** 30
Subject No.: IM60208   **Subject:**   Generalized Linear Models and Applications

Department/Center/School: Department of Industrial & Systems Engineering

Specific charts, graph paper, log book etc., required: NIL.
Special Instructions (if any):

1. Answer all questions.

2. Assume any missing values, clearly stating the assumption.

3. Some useful statistical tables at the end of the question paper.

Q.1) Short answer questions   Total: 12 pts

(a) Distinguish between Prediction Intervals and Confidence Intervals in Regression?   2 marks

(b) Define the notion of a Likelihood function?   2 marks

(c) Depict and explain the difference between the concepts of score test, Wald test, and Likelihood ratio test using the Loglikelihood curve?   3 marks

(d) What is the difference between a Cohort sampling, Clinical sampling, and Case control sampling? (You are expected to write about underlying sampling design and under what situation you would choose each of these designs?)   2 marks

(e) Examine if the Normal distribution is part of the exponential family of distributions?   3 marks

Q.2) For the following contingency table (table 1) depicting the the Political belief (Democrat/Republican) and the preferred Genre of Music (Classical,Country, Hip-Hop, Rock, Others) for a pre-decided sample size of 300 individuals. The people conducting this study wishes to analyse if the Political belief is influencing the Genre of Music preferred by an Individual?

Table 1: Contingency table between Political affiliation and Music Preference

	Classical	Country	Hip-Hop	Rock	Others
Republican	30	40	30	20	25
Democrat	40	40	25	25	25

Total: 8 pts

(a) Using odds ratio infer if republicans more likely to prefer country music than democrats?   3 marks

(b) In this context, can the use of risk difference, risk ratio, and odds ratio prove independence? Justify your answer?   2 marks

(c) Test if Political affiliation is independent of Music genre preference?   3 marks

Q.3) Consider the following data of the Advertising expenditure and the Profit (USD) given in table 2? (Note: All money values are in Million USD). Here Profit may be viewed as the response.
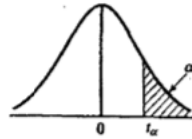
Table 2: Data for question 3

Profit	10	15	-5	-3	12	16	13
Advertising expenditure	5	6	8	4	7	10	15

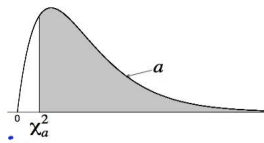For the above data, answer the following questions   **Total: 10 pts**

(a) Check if the relationship between Profit and advertising expenses are significant?   5 marks

(b) Compute the 95% Confidence interval and prediction interval for an advertising expenditure of 12 million USD?   3 marks

(c) Find the sample correlation coefficient between these variables by making using of the estimated regression equation and the R^2 value?   2 marks

v	$t_{0.005}$	$t_{0.01}$	$t_{0.025}$	$t_{0.05}$	$t_{0.10}$
1	63.66	31.82	12.71	6.31	3.08
2	9.92	6.92	4.30	2.92	1.89
3	5.84	4.54	3.18	2.35	1.64
4	4.60	3.75	2.78	2.13	1.53
5	4.03	3.36	2.57	2.02	1.48
6	3.71	3.14	2.45	1.94	1.44
7	3.50	3.00	2.36	1.90	1.42
8	3.36	2.90	2.31	1.86	1.40
9	3.25	2.82	2.26	1.83	1.38
10	3.17	2.76	2.23	1.81	1.37
11	3.11	2.72	2.20	1.80	1.36
12	3.06	2.68	2.18	1.78	1.36
13	3.01	2.65	2.16	1.77	1.35
14	2.98	2.62	2.14	1.76	1.34
15	2.95	2.60	2.13	1.75	1.34
16	2.92	2.58	2.12	1.75	1.34
17	2.90	2.57	2.11	1.74	1.33
18	2.88	2.55	2.10	1.73	1.33
19	2.86	2.54	2.09	1.73	1.33
20	2.84	2.53	2.09	1.72	1.32
21	2.83	2.52	2.08	1.72	1.32
22	2.82	2.51	2.07	1.72	1.32
23	2.81	2.50	2.07	1.71	1.32
24	2.80	2.49	2.06	1.71	1.32
25	2.79	2.48	2.06	1.71	1.32
26	2.78	2.48	2.06	1.71	1.32
27	2.77	2.47	2.05	1.70	1.31
28	2.76	2.47	2.05	1.70	1.31
29	2.76	2.46	2.04	1.70	1.31
30	2.75	2.46	2.04	1.70	1.31
40	2.70	2.42	2.02	1.68	1.30
60	2.66	2.39	2.00	1.67	1.30
120	2.62	2.36	1.98	1.66	1.29
∞	2.58	2.33	1.96	1.645	1.28

Figure 1: t-distribution table

df	$\chi^2_{0.9995}$	$\chi^2_{0.999}$	$\chi^2_{0.995}$	$\chi^2_{0.990}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.90}$	$\chi^2_{0.85}$	$\chi^2_{0.80}$
1	0.000	0.000	0.000	0.000	0.001	0.004	0.016	0.036	0.064
2	0.001	0.002	0.010	0.020	0.051	0.103	0.211	0.325	0.446
3	0.015	0.024	0.072	0.115	0.216	0.352	0.584	0.798	1.005
4	0.064	0.091	0.207	0.297	0.484	0.711	1.064	1.366	1.649
5	0.158	0.210	0.412	0.554	0.831	1.145	1.610	1.994	2.343
6	0.299	0.381	0.676	0.872	1.237	1.635	2.204	2.661	3.070
7	0.485	0.598	0.989	1.239	1.690	2.167	2.833	3.358	3.822
8	0.710	0.857	1.344	1.646	2.180	2.733	3.490	4.078	4.594
9	0.972	1.152	1.735	2.088	2.700	3.325	4.168	4.817	5.380
10	1.265	1.479	2.156	2.558	3.247	3.940	4.865	5.570	6.179
11	1.587	1.834	2.603	3.053	3.816	4.575	5.578	6.336	6.989
12	1.934	2.214	3.074	3.571	4.404	5.226	6.304	7.114	7.807
13	2.305	2.617	3.565	4.107	5.009	5.892	7.042	7.901	8.634
14	2.697	3.041	4.075	4.660	5.629	6.571	7.790	8.696	9.467

Figure 2: χ^2-distribution table